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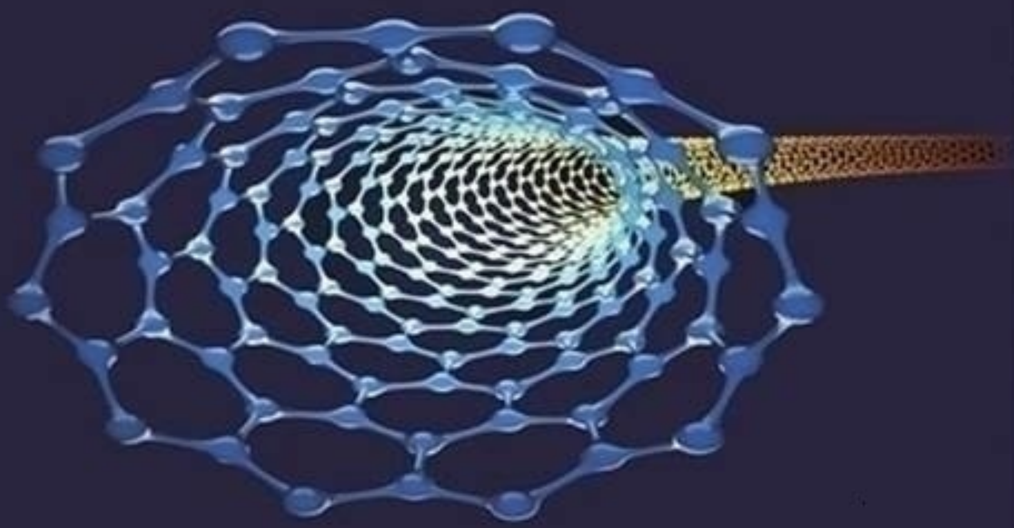

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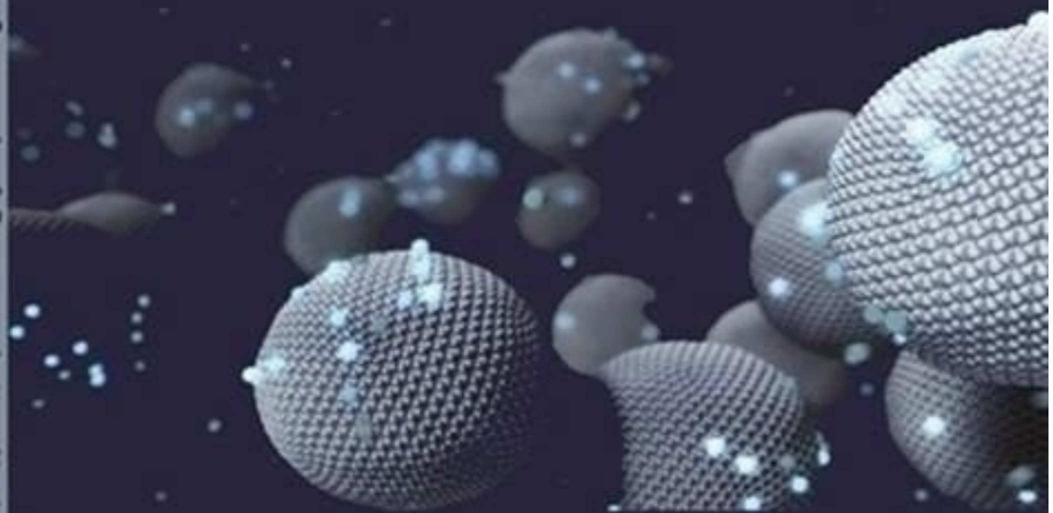

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RESEARCH PAPER

New Applications of Coding Theory in The Projective Space of Order Three

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A B S T R A C T:

The main aim of this paper is to introduce the relationship between the topic of coding theory and the projective space in field three and test the code. The maximum value of size of code over finite field of order three and an incidence matrix with the parameters, *n* (length of code), *d* (minimum distance of code) and *e* (error-correcting of code) have been constructed. With a theorem and a result that test the code if it is perfect or not.

KEY WORDS: Coding Theory, Projective Space, finite field.
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1. INTRODUCTION:

In 2018 Al-Saraji and Al-Hamidi [5],[6], applied the coding theory to the projective plane of field 3., so I did this expanding the work of Al-Saraji, where the coding theory was applied to field 3 in projective space, and there were several differences. In order to expand the work further and as a new work.

2. Coding Theory in The PG (3,3):

Theorem 1.1: [3],[4]

Every line in PG (3,3) contains exactly q+1 points.

Theorem 1.2:[9]

Every plane in PG (3,3) contains exactly q^2+ q+1 point (line).

Theorem 1.3: [10]

There exist q^3+q^2+ q+1 points in PG(3,3).

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Theorem 1.5:[6]

(n, M, d)-code C satisfies

M $\{\binom{n}{0}+\binom{n}{1}(q\text{-}1)+\ldots+\binom{n}{e}(q-1)^{e}\}\leq q^{n}$

Corollary 1.6:[6]

(n, M ,d)-code C is perfect if and only if M $\{\binom{n}{0}+\binom{n}{1}(q\text{-}1)+\ldots+\binom{n}{e}(q-1)^{e}\}=q^{n}$

Now let's talk about the fields of order three of the projective space of the three -dimensions. And let us have the following points generated from a generated matrix: $\{T_1$=[1,0,0,0], T_2=[0,1,0,0], T_3=[1,1,0,0], T_4=[2,1,0,0], T_5=[0,1,1,0], p_6=[1,1,1,0], T_7=[2,1,1,0], T_8=[0,2,1,0], T_9=[0,0,1,0], T_{10}=[1,0,1,0], T_{11}=[2,0,1,0], T_{12}=[1,2,1,0], T_{13}=[2,2,1,0], T_{14}=[0,0,0,1], T_{15}=[1,0,0,1], T_{16}=[2,0,0,1], T_{17}=[0,1,0,1], T_{18}=[1,1,0,1], T_{19}=[2,1,0,1], T_{20}=[0,2,0,1], T_{21}=[1,2,0,1], p_{22}=[2,2,0,1], T_{23}=[0,0,1,1], T_{24}=[1,0,1,1], T_{25}=[2,0,1,1], T_{26}=[0,1,1,1], T_{27}=[1,1,1,1], T_{28}=[2,1,1,1], T_{29}=[0,2,1,1], T_{30}=[1,2,1,1], T_{31}=[2,2,1,1], T_{32}=[0,0,2,1], T_{33}=[1,0,2,1], T_{34}=[2,0,2,1], T_{35}=[0,1,2,1], T_{36}=[1,1,2,1], T_{37}=[2,1,2,1], T_{38}=[0,2,2,1], T_{39}=[1,2,2,1], T_{40}=[2,2,2,1] $\}$

Table of planes for field 3:

π_1	π_2	π_3	π_4	.	.	.	π_{37}	π_{38}	π_{39}	π_{40}
2	1	4	3	.	.	.	3	1	3	4
5	9	7	6	.	.	.	5	8	7	6
8	10	9	9	.	.	.	11	12	8	8
9	11	12	13	.	.	.	12	13	10	11
14	14	14	14	.	.	.	15	17	16	15
17	15	19	18	.	.	.	19	18	17	17
20	16	21	22	.	.	.	20	19	21	22
23	23	23	23	.	.	.	23	23	23	23
26	24	28	27	.	.	.	27	24	27	28
29	25	30	31	.	.	.	31	25	31	30
32	32	32	32	.	.	.	34	38	33	34
35	33	37	36	.	.	.	35	39	37	36
38	34	39	40	.	.	.	39	40	38	38

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It is obvious that space contains a table of lines and plans and a table of planes now we will create a table for the planes and planes that will be obtained according to the following equation: $x_1\, y_1 + x_2\, y_2 + x_3\, y_3 + x_4\, y_4 = 0$, when $T_i = [x_1, x_2, x_3, x_4]$ and $T_j = [y_1, y_2, y_3, y_4]$

1	2	5	8	9	14	17	20	23	26	29	32	35	38
	5	14	14	14	2	5	5	2	8	9	2	9	8
	8	26	29	23	17	29	23	26	20	20	35	17	17
	9	38	35	32	20	32	35	29	32	38	38	26	23
2	1	9	10	11	14	15	16	23	24	25	32	33	34
	9	14	14	14	1	9	9	1	16	10	1	10	11
	10	23	24	25	15	24	25	24	11	15	33	16	15
	11	32	34	33	16	33	34	25	32	32	34	23	23
.	.	.	.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.	.	.	.
40	4	6	8	11	15	17	22	23	28	30	34	36	38
	6	15	17	17	4	6	6	4	8	11	11	8	4
	8	28	23	28	17	30	23	28	22	22	15	15	34
	11	38	38	36	22	34	36	30	34	38	23	30	36

Theorem 1.7

The projective space of in field three is a code with a parameters $[n = 40, d=13, e=6, M=3^{36}]$

proof: The space $\pi_3$has an incidence matrix $A=(a_{ij})$,where

$a_{ij} = 1$ if $T_j \in \pi_i$ And also condition $a_{ij} = 0$ if $T_j \notin \pi_i$

After applying the theorem we will have a table of the planes on which the point lie and which do not and in field 3we have 0,1,2 where z=[0,0,0,0,0,0,0,0,0,0,0,0,0,0,0…,0,0,0,0,0] and u=[1,1,1,1,1,1…,1,1,1,1,1,1,1,1,1] and w =[2,2,2,2,2,2,2,2,2,2,2…,2,2,2,2,2,2,2] and then we have the table m from the law $m_i = u + l_i$ and table V from $v_i = w + l_i$ and then find these values $d(z,\pi_i)$and $d(u,\pi_i), d(w,\pi_i), d(\pi_i,\pi_j), d(m_i,\pi_i), d(\pi_i,v_i), d(z,m_i), d(u,m_i), d(w,m_i), d(m_i,m_j), d(m_i,v_j), d(z,v_i), d(u,v_i), d(w,v_i)$,$d(v_i,v_j)$ and $i \neq j$ in the following tables of the planes .

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	T_1	T_2	T_3	T_4	.	.	.	T_{37}	T_{38}	T_{39}	T_{40}
$\boldsymbol{\pi_1}$	0	1	0	0	.	.		0	1	0	0
$\boldsymbol{\pi_2}$	1	0	0	0	.	.		0	0	0	0
$\boldsymbol{\pi_3}$	0	0	0	1	.	.		1	0	1	0
$\boldsymbol{\pi_4}$	0	0	1	0	.	.		0	0	0	1
.	.	.	.	.	.	.		.	.	.	.
.	.	.	.	.	.	.		.	.	.	.
.	.	.	.	.	.	.	.	.	.	.	.
$\boldsymbol{\pi_{37}}$	0	0	1	0	.	.	.	0	0	1	0
$\boldsymbol{\pi_{38}}$	1	0	0	0	.	.	.	0	1	1	1
$\boldsymbol{\pi_{39}}$	0	0	1	0	.	.	.	1	1	0	0
$\boldsymbol{\pi_{40}}$	0	0	0	1	.	.	.	0	1	0	0

Now the table of m_i

$\mathbf{m_1}$	1	2	1	1	.	.	.	1	2	1	1
$\mathbf{m_2}$	2	1	1	1	.	.	.	1	1	1	1
$\mathbf{m_3}$	1	1	1	2	.	.	.	2	1	2	1
$\mathbf{m_4}$	1	1	2	1	.	.	.	1	1	1	2
.	.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.	.
$\mathbf{m_{37}}$	1	1	2	1	.	.	.	1	1	2	1
$\mathbf{m_{38}}$	2	1	1	1	.	.	.	1	2	2	2
$\mathbf{m_{39}}$	1	1	2	1	.	.	.	2	2	1	1
$\mathbf{m_{40}}$	1	1	1	2	.	.	.	1	2	1	1

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The table of v_i

v_1	2	0	2	2	.	.	.	2	0	2	2
v_2	0	2	2	2	.	.	.	2	2	2	2
v_3	2	2	2	0	.	.	.	0	2	0	2
v_4	2	2	0	2	.	.	.	2	2	2	0
.	.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.	.
v_{37}	2	2	0	2	.	.	.	2	2	0	2
v_{38}	0	2	2	2	.	.	.	2	0	0	0
v_{39}	2	2	0	2	.	.	.	0	0	2	2
v_{40}	2	2	2	0	.	.	.	2	0	2	2

these values $d(z,\pi_i)=13$ and $d(u,\pi_i)=27$,$d(w,\pi_i)=40$,$d(\pi_i,\pi_j)=18$,$d(m_i,\pi_i)=40$ $d(\pi_i,v_i)=40$,$d(z,m_i)=40$,$d(u,m_i)=13$,$d(w,m_i)=27$,$d(m_i,m_j)=18$,$d(m_i,v_i)=40$,$d(z,v_i)=27$,$d(u,v_i)=40$,$d(w,v_i)=13$,$d(v_i,v_j)=18$.If we substitute the values of $n = 40$,$d= 13$,$e =6$,Hence C is a$(40,3^{36},13)$-code .And applying the theorem 2.2.3 we gets that :

$$3^{36}\{\binom{40}{0}+\binom{40}{1}(2)+\binom{40}{2}(4)+\binom{40}{3}(8)+\binom{40}{4}(16)+\binom{40}{5}(32)+\binom{40}{6}(64)\}$$

$$= 3^{36}(1+80+3120+79040+1462240+21056256+245656320)$$

$$= 3^{36}(1+80+3120+79040+1462240+21056256+245656320)$$

$\neq q^n$, $n = 40$ By corollary 1.6 therefore C is not perfect

3. CONCLUSIONS

The application of the coding theory to certain fields in space and plane, we summarize them in the following table:

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