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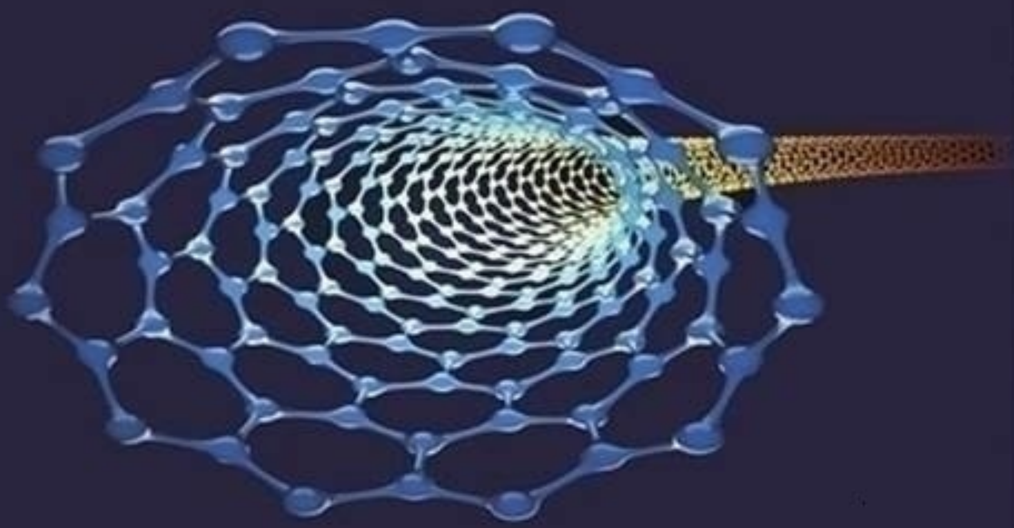

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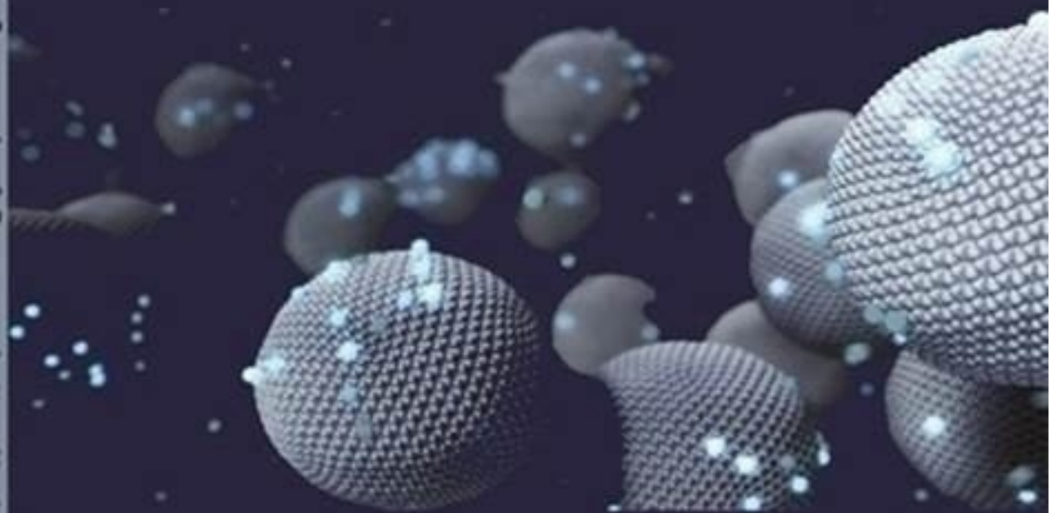

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RESEARCH PAPER

A New Transformation Technique to Solve Multi-Objective Linear Programming Problems

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A B S T R A C T:

In this paper Standard Error of Mean (SEM), as a new technique, is used for transforming multi-objective linear programming problems (MOLPPs) to the single objective linear programming problems (SOLPPs). To this end, an algorithm has been proposed and suggested to solve MOLPPs, which have been tested through numerical examples by employing Excel Solver. However, the study compares the results of other techniques like (Chandra Sen, Optimal Average of Minimax and Maximin, New Arithmetic Average, New Geometric Average, New Harmonic Average, and Advanced Transformation) with the results of this new technique SEM. The numerical results indicate that a new technique in general is promising.

KEY WORDS: Multi-Objective Linear Programming, Geometric Average, Harmonic Average.
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1. INTRODUCTION:

A single objective function has frequently been optimized (maximized or minimized) using linear programming under certain restrictions. It can be challenging to maximize two or more objectives at once, and this challenge increases if the objectives are incompatible. It was decided to investigate the potential of coming up with a compromise solution that meets all the objectives. Many techniques have been utilized for the purpose of resolving multi-objective optimization problems. In 1983, [1] was the first to introduce the problem of multi-objective linear programming. Following that, several alternative statistical methods were developed that are now followed by numerous intellectuals and specialists.

Many strategies have been presented, particularly in the last four decades. For example, [2] proposed an approach for solving multi-objective fractional programming problems. The algorithm's computer application was tested on a variety of numerical problems, and the technique was improved by employing mean and median values for objective function values [3].

To tackle the challenges of the multi-objective programming, the Chandra Sen method has been enhanced since then. [4] proposed a new approach based on the optimum average technique that was a development of the Chandra Sen [1] and [3] techniques. Moreover, regarding mean and median, [5] offered the harmonic average (mean) technique to turn MOLPPs into single-objective linear programming problems.

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[6] also proposed a novel strategy for solving multi-objective linear fractional programming issues utilizing a novel geometric averaging method that outperformed other methods. The newly suggested harmonic averaging approach outperforms all other averaging methods in terms of performance. [7] developed an enhanced transformational strategy for solving MOPP, claiming that the strategy was very simple to compute.

To expand on the previous work, the current research is an attempt to solve the MOLPP as a single linear programming problem using an unproven approach called standard error of mean. Because of its more reasonable and efficient implementations, multi-objective programming has gained popularity among many scholars. The present work describes a MOLPP and proposes a standard error of mean approach to optimize the objective function, where a single objective function is formed from multi-objective functions. The outcome is compared to that of optimization utilizing average and the most current methodologies by implementing Excel solver. The new approach used in the present work and its results outperforms the outcomes of all the other aforementioned techniques.

2. DEFINITIONS:

MULTI-OBJECTIVE LINEAR PROGRAMMING PROBLEMS

The mathematical form of MOLPP is given as follows:

$Max\ \mathcal{K}_1 = C_1^t x$
$Max\ \mathcal{K}_2 = C_2^t x$
$\vdots$
$Min\ \mathcal{K}_{r+1} = C_{r+1}^t x$
$\vdots$
(1)
$Min\ \mathcal{K}_s = C_s^t x$
s/t :-
$AX\ (\geq, =, or \leq)\ b$
$X \geq 0$

where, b is m-dimensional vector of constants, x is n dimensional vector of decision variables and A is $m \times n$ matrix of constants.

3. ARITHMETIC AVERAGE [8]:

Arithmetic average (AA) which is sometimes referred to as mean, is the most commonly used central value of a distribution. The AA is calculated by the sum of the values of all the observations $y_1, y_2, \dots, y_n$ and this total is divided by the number of observations: AA = $\frac{y_1+y_2+\cdots+y_n}{n}$, where n is the total number of observations.

4. GEOMETRIC AVERAGE [9]:

The geometric average (GA) of n positive observation values $y_1, y_2, \dots, y_n$ is defined as the nth positive root of the product of the values: GA = $\sqrt[n]{y_1 y_2 \dots y_n}$, where n is the total number of observations.

5. HARMONIC AVERAGE [9]:

The harmonic average (HA) is another type of averages of observation values $y_1, y_2, \dots, y_n$: HA= $\frac{n}{\frac{1}{y_1}+\frac{1}{y_2}+\cdots+\frac{1}{y_n}}$, where n is the total number of observations.

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6. STANDARD ERROR OF MEAN [10]:

The standard error of mean (SEM) of n positive observation values $y_1, y_2, \dots, y_n$ is defined mathematically as the standard deviation (SD) of $y_1, y_2, \dots, y_n$ that is divided by the square root of n; $SEM = \frac{SD}{\sqrt{n}}$, where standard deviation SD $=\sqrt{\frac{\sum_{i=1}^{n}(y_i - \mathrm{AA})^2}{n-1}}$, and AA is the arithmetic average of $y_1, y_2, \dots, y_n$.

7. APPLIED TECHNIQUES OF CONVERTING MULTI-OBJECTIVE OPTIMIZATION PROBLEMS INTO SINGLE OPTIMIZATION PROBLEM:

Various strategies are presented in the literature to resolve the MOLPP. Initially, Chandra Sen's approach was employed to solve multi-objective optimization problems, yielding a somewhat poor outcome for the objective function. Then, to solve it, additional known and recommended strategies are applied. These strategies are concisely detailed below.

8. CHANDRA SEN'S (CS) TECHNIQUE [1]:

Using simplex method to solve MOLPP in equation (1), a single value corresponding to each of the objective functions is obtained which are in equation (2).

$Max\ \mathcal{K}_1 = Л_1$
$Max\ \mathcal{K}_2 = Л_2$
$\vdots$
(2)
$Max\ \mathcal{K}_r = Л_r$
$Min\ \mathcal{K}_{r+1} = Л_{r+1}$
$\vdots$
$Min\ \mathcal{K}_s = Л_s$

Where $Л_1$, $Л_2$,…, $Л_s$ are values of the objective functions. These values are utilized in Chandra Sen's technique to obtain a single objective function as shown in equation (3).

$$\text{Max } \mathcal{K} = \sum_{i=1}^{r} \frac{\mathcal{K}_i}{|Л_i|} + \sum_{i=r+1}^{s} \frac{\mathcal{K}_i}{|Л_i|}, \tag{3}$$

where $Л_i \neq 0$, i=1, …,s subject to the constraints of equation (1), and the optimum value of the objective functions $Л_i$ may be positive or negative.

9. OPTIMAL AVERAGE OF MINIMAX AND MAXIMIN (OAXN)TECHNIQUE [11]:

m_1 =min { $|Л_i|$ }, i = 1, 2, …, r.
m_2 =max { $|Л_i|$ }, i = r + 1, …, s.
OAXN = $\frac{m_1+m_2}{2}$. The MOLPP becomes Max $\mathcal{K} = \frac{\sum_{i=1}^{r} Max\, \mathcal{K}_i - \sum_{i=r+1}^{s} Min\, \mathcal{K}_i}{OAXN}$.

10. NEW ARITHMETIC AVERAGE (NAA) TECHNIQUE [12]:

m_1 =min { $|Л_i|$ }, i = 1, …, r.
m_2 =min { $|Л_i|$ }, i = r + 1, …, s.
NAA = $\frac{m_1+m_2}{2}$. The MOLPP becomes Max $\mathcal{K} = \frac{\sum_{i=1}^{r} Max\, \mathcal{K}_i - \sum_{i=r+1}^{s} Min\, \mathcal{K}_i}{\mathrm{NAA}}$.

11. NEW GEOMETRIC AVERAGE (NGA) TECHNIQUE [12]:

m_1 =min { $|Л_i|$ }, i = 1, …, r.
m_2 =min { $|Л_i|$ }, i = r + 1, …, s.
NGA= $\sqrt{m_1 m_2}$. The MOLPP becomes Max $\mathcal{K} = \frac{\sum_{i=1}^{r} Max\, \mathcal{K}_i - \sum_{i=r+1}^{s} Min\, \mathcal{K}_i}{\mathrm{NGA}}$.

12. NEW HARMONIC AVERAGE (NHA) TECHNIQUE [7]:

m_1 =min { $|Л_i|$ }, i = 1, …, r.
m_2 =min { $|Л_i|$ }, i = r + 1, …, s.

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NHA=$\frac{2}{\frac{1}{m_1}+\frac{1}{m_2}}$. The MOLPP becomes Max $\mathcal{K} = \frac{\sum_{i=1}^{r} Max\, \mathcal{K}_i - \sum_{i=r+1}^{s} Min\, \mathcal{K}_i}{NHA}$.

13. ADVANCED TRANSFORMATION (AT) TECHNIQUE [7]:

m_1 =min { $|Л_i|$ }, $i = 1, \ldots, r$.
m_2 =min { $|Л_i|$ }, $i = r + 1, \ldots, s$.
m= min { m_1, m_2 }. The MOLPP becomes Max $\mathcal{K} = \frac{\sum_{i=1}^{r} Max\, \mathcal{K}_i - \sum_{i=r+1}^{s} Min\, \mathcal{K}_i}{m}$.

14. NEW TRANSFORMATION TECHNIQUE: STANDARD ERROR OF MEAN (SEM) TECHNIQUE:

The standard error of mean technique is a proposed technique which is used to come up with a single optimization problem from multi-objective optimization problems. It is denoted by $EM = \frac{SD}{\sqrt{s}}$, SD is the standard deviation of $|Л_i|$, $\forall\ i = 1, \cdots, s$; SD=$\sqrt{\frac{\sum_{i=1}^{s}(|Л_i|-\text{AA})^2}{s-1}}$, and AA is the arithmetic average of $|Л_i|$, $\forall$ $i = 1, \cdots, s$.
The MOLPP becomes Max $\mathcal{K} = \frac{\sum_{i=1}^{r} Max\, \mathcal{K}_i - \sum_{i=r+1}^{s} Min\, \mathcal{K}_i}{SEM}$, subject to the same constraints.

3. ALGORITHM:

The following algorithm is followed to obtain the optimal solution for the MOLP problems defined previously via using SEM technique can be summarized through the following steps:

1. Find the value of optimal solution of all the objectives individually by simplex method via Excel solver, which is to be maximized or minimized.
2. Calculate the SEM of the optimal values; $|Л_i|$, $i = 1, \ldots, s$.
3. Construct the combined objective function by subtracting weighted minimization objective function from weighted maximization objective function; $\sum_{i=1}^{r} Max\, \mathcal{K}_i - \sum_{i=r+1}^{s} Min\, \mathcal{K}_i$.
4. Divide the result of step (3) by SEM; $\frac{\sum_{i=1}^{r} Max\, \mathcal{K}_i - \sum_{i=r+1}^{s} Min\, \mathcal{K}_i}{SEM}$.
5. Solve the result (4) by simplex method via employing Excel solver, then the solution of the problem is obtained.

4. NUMERICAL EXAMPLES:

Consider the following multi-objective linear programming problems. The following two examples are solved for the analysis purposes of the present study; the results of the two examples are also compared for the purposes of making a distinction between CS, OAXN, NAA, NGA, NHA and AT techniques and SEM as a more effective technique.

EXAMPLE 1

$$\begin{gathered}
\text{Max } \mathcal{K}_1 = 3x_1 + 2x_2 \\
\text{Max } \mathcal{K}_2 = 4x_1 + x_2 \\
\text{Max } \mathcal{K}_3 = 4x_1 - 2x_2 \\
\text{Max } \mathcal{K}_4 = 15x_1 + 4x_2 \\
\text{Min } \mathcal{K}_5 = -6x_1 + 2x_2 \\
\text{Min } \mathcal{K}_6 = -9x_1 + 3x_2 \\
\text{Min } \mathcal{K}_7 = -5x_1 + 2x_2 \\
\text{s/to: -} \\
x_1 + x_2 \le 4 \\
x_1 - x_2 \le 2 \\
x_1, x_2 \ge 0
\end{gathered}$$

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SOLUTION:

After finding the value of each of the individual objective functions by simplex method employing Excel solver; the results are as below in Table 1:

TABLE 1: Results of Example 1

i	$\mathcal{K}_i$	(x_1, x_2)	$\lvert\mathcal{K}_i\rvert$
1	11	(3,1)	11
2	13	(3,1)	13
3	10	(3,1)	10
4	49	(3,1)	49
5	-16	(3,1)	16
6	-24	(3,1)	24
7	-13	(3,1)	13

EXAMPLE 2

$$\text{Max } \mathcal{K}_1 = 0.5x_1 + 0.66x_2 + 0.833x_3$$
$$\text{Max } \mathcal{K}_2 = 0.25x_1 + 0.33x_2 + 0.415x_3$$
$$\text{Min } \mathcal{K}_3 = 0.2x_1 - 0.34x_2 - 0.3x_3$$
$$\text{Min } \mathcal{K}_4 = 0.3x_1 - 0.32x_2 - 0.32x_3$$
$$\text{s/to: -}$$
$$3x_1 + 4x_2 + 2x_3 \leq 60$$
$$2x_1 + x_2 + 2x_3 \leq 40$$
$$x_1 + 3x_2 + 2x_3 \leq 80$$
$$x_1, x_2, x_2 \geq 0$$

SOLUTION:

After finding the value of each of the individual objective functions by simplex method employing Excel solver, the results are as below in Table 2:

TABLE 2: Results of Example 2

i	$\mathcal{K}_i$	(x_1, x_2, x_3)	$\lvert\mathcal{K}_i\rvert$
1	18.283	(0,6.667,16.667)	18.283
2	9.12	(0,6.667,16.667)	9.12
3	-7.27	(0,6.667,16.667)	7.27
4	-7.47	(0,6.667,16.667)	7.47

5. RESULTS AND DISCUSSION:

Table 3 summarizes the solutions of the MOLPP using techniques. It shows that the solution of the objective functions is improved when used the proposed new transformation technique SEM is used.

TABLE 3. Comparison between the results of techniques

Techniques	**Example1**				**Example2**			
	$\mathcal{K}$	(x_1, x_2)	**Time**	N. Iterations	$\mathcal{K}$	(x_1, x_2, x_3)	**Time**	N. Iterations

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CS	7	(3,1)	0.062	2	3.999	(0, 6.667, 16.667)	0.063	2
OAXN	8	(3,1)	0.063	2	5.077	(0, 6.667, 16.667)	0.063	2
NAA	11.826	(3,1)	0.062	2	5.141	(0, 6.667, 16.667)	0.078	2
NGA	11.928	(3,1)	0.047	2	5.174	(0, 6.667, 16.667)	0.078	2
NHA	12.031	(3,1)	0.078	2	5.208	(0, 6.667, 16.667)	0.063	2
AT	13.6	(3,1)	0.078	2	5.796	(0, 6.667, 16.667)	0.062	2
SEM	**23.7**	(3,1)	**0.047**	**2**	**16.114**	(0, 6.667, 16.667)	**0.078**	**2**

It can be seen that in Example 1, the values of x_1, x_2 and the number of iterations do not change in all the techniques; however, the value of objective functions is improved and increased in the techniques successively. As Table 3 shows, the difference in the value of $\mathcal{K}$ is significantly higher using the SEM technique compared to all the previously used techniques. This indicates that the SEM technique is more effective than such techniques as CS, OAXN, NAA, NGA, NHA and AT. There is time difference in solving each problem using each technique as presented in the table. The CS and OAXN techniques needed 0.062 seconds only for the solution of the problem. However, for the transformation of the Example 1, the maximum time required is 0.078 seconds for the NHA and AT techniques; their results are better than OAXN, NAA, NGA and SEM. Interestingly, although the value of $\mathcal{K}$ differs in NGA and SEM techniques, with 11.928 and 23.7, respectively- the required time for solving is the same in the first example. Thus, the highest value of $\mathcal{K}$ does not take the least time.

In Example 2, the less consuming time technique goes for the AT, whereas the most consuming time is needed for NAA, NGA and SEM but with the optimize value 5.141, 5.174 and 16.114, respectively. This reveals that in Excel solver, the required time changes depending on the technique being used and ensures that two different techniques with big differences of optimal values may take the same time. Just like Example 1, in Example 2 also the results of x_1, x_2, x_3 and the number of iterations remain the same in all the techniques used in the present study. The result of SEM in Example 2 proves to be better than all the other techniques, namely, CS, OAXN, NAA, NGA, NHA and AT.

6. CONCLUSION AND FUTURE WORK:

Different techniques such as CS, OAXN, NAA, NGA, NHA and AT are used to solve MOLPP, and the results are compared in Table 3. As a new transformation technique, SEM is used to transform MOLPP into single linear programming problem. As the obtained results reveal, for transforming MOLPP, SEM is better suited for optimizing MOLPP compared to such techniques as CS, OAXN, NAA, NGA, NHA and AT.

The results show the importance of the proposed SEM as a new transformation technique for the optimizing MOLPP using standard error of mean for the very first time. In Example 1, the consumed time for NGA and SEM is the same although value of $\mathcal{K}$ is remarkably different with 11.928 and 23.7, respectively. Similarly, for solving Example 2, NAA, NGA and SEM takes the same time with 0.078 seconds only despite the great difference in their $\mathcal{K}$ value with 5.141, 5.174 and 16.114, respectively. This reveals that whether the value of $\mathcal{K}$ is greater or not, the required time to solve the MOLPPs remains the same. It cannot be generalized that the greater the value of $\mathcal{K}$ is, the more or less time it will require to solve.

The future studies should compare SEM with techniques other than CS, OAXN, NAA, NGA, NHA and AT to reinforce the results and come up with better results.

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